

Overview of Scales

While preparing for scales, the following steps are to be followed:

1) What is the **Representative fraction (R.F)** of the problem?

(A) The **R.F** is calculated by the following formula:

$$R.F = \frac{\text{Length of the object on drawing sheet (in cm)}}{\text{Actual length of the object (in cm)}}$$

This may be given in a statement form as shown below:

- (i) On a map, 1 cm represents 10 meters of actual length. Find the RF?
- (ii) An area of 144 sq cm on a map represents an area of 36 sq km on the field. Find the RF?
- (iii) A room of 1728 m³ volume is shown as 216 cm³ volume on a drawing sheet. What is the RF?

For calculating RF, always consider the linear dimension only. If **square units** are mentioned in dimensions, find the **square root** and if **cubes** (volumes) are mentioned, find the **cube root** of the ratio to get the RF.

E.g.: for (i), $RF = \frac{1 \text{ cm}}{10 \text{ m}} = \frac{10 \text{ mm}}{10 \text{ m} \times 1000 \text{ (in mm)}}$
 $= \mathbf{1/1000}.$

For (ii), $RF = \sqrt{\frac{144 \text{ cm}^2}{(36 \times (1000 \times 100)^2 \text{ cm}^2)}}$
 $= \frac{12}{360000}$
 $= \mathbf{1/50000}$

For (iii), $RF = \sqrt[3]{\frac{216 \text{ cm}^3}{1728 \times (100)^3 \text{ cm}^3}}$
 $= \mathbf{1 / 200}.$

2) Maximum Length (M.L):

This data is usually given in the problem. It will be in terms of maximum length that has to be measured by the given scale. It is taken as M.L

- E.g. Construct a scale to measure up to 5 meters → M.L = 5m.
 Construct a scale to read up to 10 meters → M.L = 10m.
 Construct a scale to read up to 600 meters → M.L = 600m.

3) Length of Scale (LOS) (always in cm or mm) :

LOS is the actual length of the scale that is drawn for the given problem.

LOS is calculated by the following formula:

$ \begin{aligned} LOS &= R.F \times \text{Maximum Length} \\ &= R.F \times M.L \quad (\text{in cm or mm}) \end{aligned} $
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- E.g. (i) For a plain scale, RF = 1/4 and M.L = 5 decimeters, Find LOS.

$$\begin{aligned}
 LOS &= \frac{1}{4} \times 5 \times 10 \text{ cm (Since 1 dm = 10 cm)} \\
 &= 12.5 \text{ cm.}
 \end{aligned}$$

The length of the line that is drawn on the drawing sheet is 12.5 cm.

- (ii) For a diagonal scale, RF = 3/200 & M.L = 6 meters. Calculate the L.O.S

$$\begin{aligned}
 LOS &= \frac{3}{200} \times 6 \times 100 \text{ cm (Since 1 m = 100 cm)} \\
 &= 9 \text{ cm.}
 \end{aligned}$$

The length of the line that is drawn on the drawing sheet is 9 cm.

4) Division of LOS into n equal parts (n is Maximum Length ML)

After drawing a line of length equal to LOS, the next step is to divide the line into a certain no. of equal parts. This is found by checking the Maximum Length.

→ Case 1: When ML < 10:

E.g. If **LOS = 12.5 cm** and **ML = 5 dm**, then **Divide LOS into 5 equal parts** so that each part represents **1 dm**.

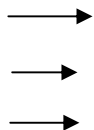
If **LOS = 15 cm** and **ML = 800 meters**, **divide LOS into 8 equal parts** so that each part represents **100 meters** (by line division method)

→ Case 2: When ML > 10:

If ML in the given problem is greater than 10, then we have to take the factors of the ML and divide the LOS.

Eg1. If **ML = 25m**, then we can't divide the LOS into 25 parts. Hence take the factors of 25, i.e. 5×5 . Hence divide the LOS into 5 equal parts so that each part represents 5m.

Thus, If factors of ML are **m x n**, then divide ML into **m equal parts** of **n units** each.



Eg 2. If **LOS = 21 cm** and **ML = 42 meters (7 x 6)**, **divide LOS into 7 equal parts** so that each part represents **6 meters** or **divide LOS into 6 equal parts** so that each part represents **7 meters**.

Sometimes, the ML will be given in terms of a non related scale units. Then, we need to convert the ML into the units used in the scale and then divide LOS into ML no.of parts.

Eg 3: Draw a plain scale to show decimeters & centimeters and to measure upto 1m.

Sol: Here, we know that a plain scale relates only a main unit and its immediate sub unit. Thus, after dm, cm is its sub unit. So, meter is a unit not required in the scale. But ML is given in meters. So it has to be converted into dm since dm is the main scale.

Thus, $ML = 1 \text{ m} = \underline{10} \text{ dm}$. Hence, the LOS can be divided into 10 equal parts.

Eg 4: Draw a diagonal scale to show meters and measure upto 1 km.

Sol: $1 \text{ km} = 1000 \text{ mts}$ (10×100). Hence ML is divided into 10 equal parts of 100 m each.

5) Division of the first part into sub-divisions:

The first part of all the scales is always divided into sub-divisions depending on the main unit and the sub units.

Relation between various units and sub units:

1 kilometer	=	10 hecta meters.	(1 Km	=	10 Hm)
1 hecta meter	=	10 deca meters.	(1 Hm	=	10 Dm)
1 deca meter	=	10 meters.	(1 Dm	=	10 m)
1 meter	=	10 decimeters	(1 m	=	10 dm)
1 deci meter	=	10 centi meters	(1 dm	=	10 cm)
1 centi meter	=	10 milli meters	(1 cm	=	10 mm)
1 furlong	=	200 meters				
1 Mile	=	1.6 km	=	1600 meters	=	8 Furlongs
1 yard	=	3 feet (3')				
1 foot (1')	=	12 inches (12'');		1 inch (1'')	=	2.54 cm

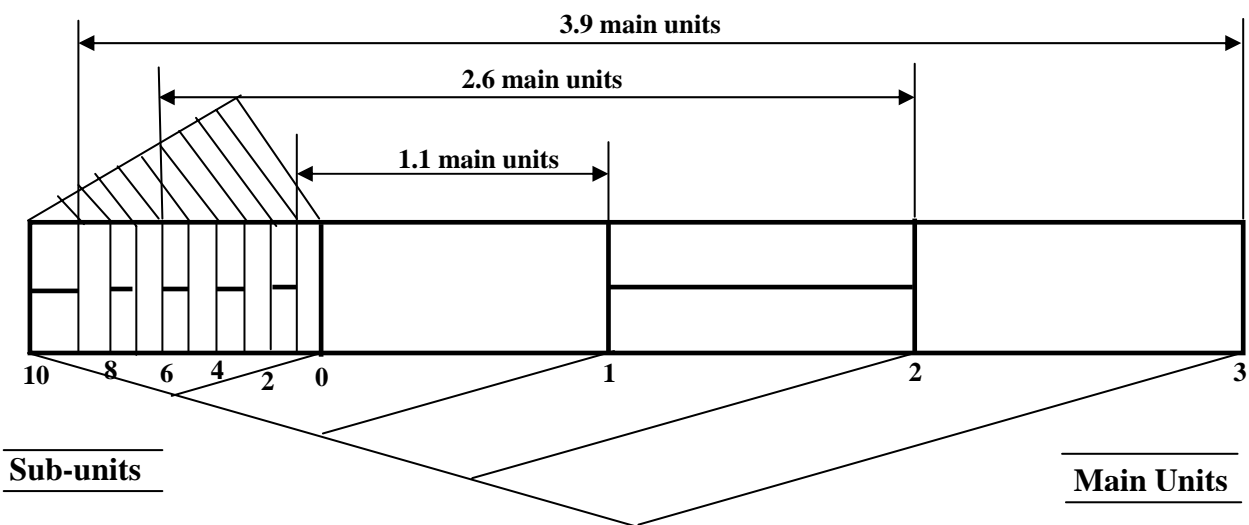
Plain Scales

Plain scales are used for measurement in two units or a unit and its sub unit. It consists of a line (LOS) divided into suitable no. of equal parts (based on Max Length, ML) & the first part is sub-divided into equal parts.

The zero is placed at the end of 1st main division. Labeling of 0 in main scale is after the first unit.

Steps involved in Plain Scales:

- 1) Find **RF**.
- 2) Find **Maximum Length (ML)**.
- 3) Find **Length of Scale (LOS)** by using the formula. (**LOS = RF X ML**).
- 4) **Draw the line of length = LOS & divide it into n equal parts** based on **ML**. Draw a **rectangle on LOS of height 5 mm**.
- 5) **Divide the first division also into some equal parts** based on the unit and its sub-unit given in the problem.
- 6) **Give numbering to main scale as 0,1,2,3,...** after the first division to the **right side** & 0,1,2,3... to the **left of 0 in 1st sub division**.
- 7) Mark the given value of dimension based on problem.



$$\text{R.F} = \frac{1}{100}$$

Problems on Plain Scales:

As solved examples are explained well in the text book (ND Bhat), only exercise problems are presented in this notes.

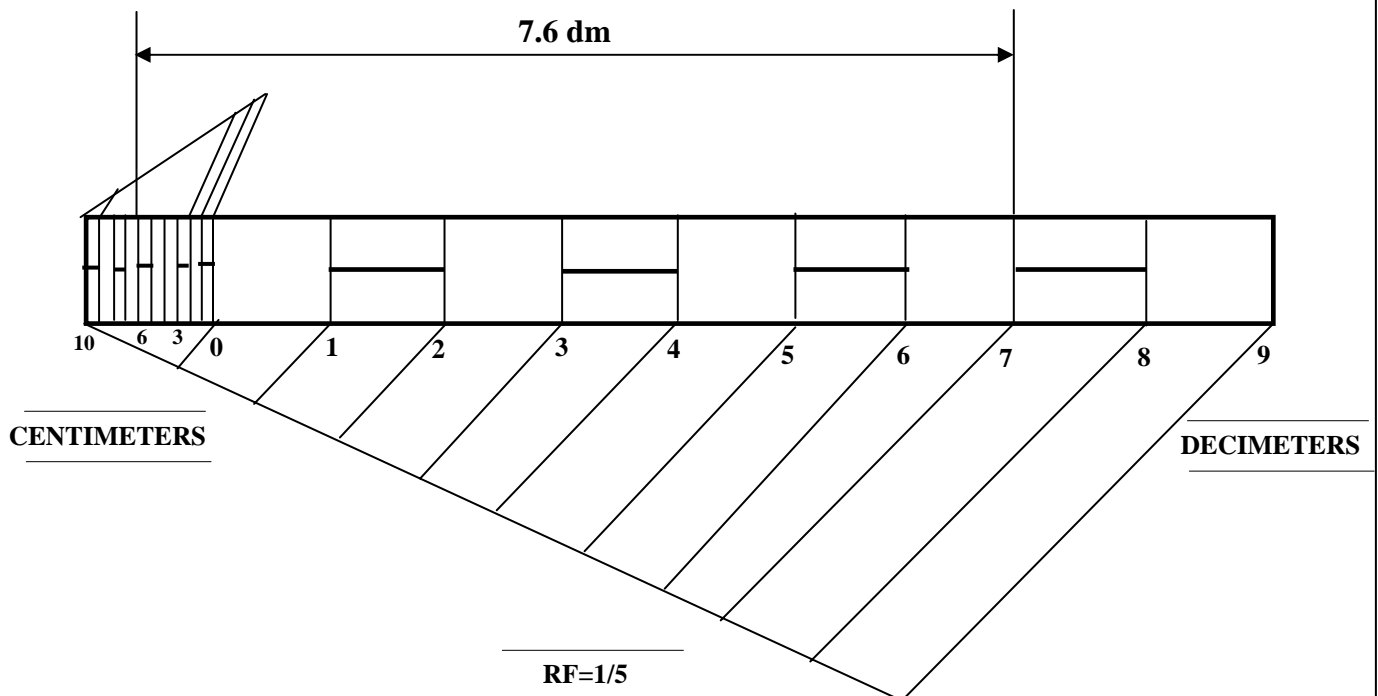
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Q2) Construct a scale of **1:5** to show **decimeters** and **centimeters** and to read up to **1 meter**. Show the length of **7.6 dm** on it.

Sol) Given data:

$$\begin{aligned}
 \text{RF} &= 1/5 \\
 \text{Max Length (ML)} &= 10 \text{ dm (1 m = 10 dm) ; (no. of parts of scale (n) = 10 parts)} \\
 \text{Length of scale (LOS)} &= \text{RF} \times \text{ML} \\
 &= 1/5 \times 10 \times 10 \text{ cm (1 dm = 10 cm)} \\
 &= 20 \text{ cm.}
 \end{aligned}$$

The length of the line that is drawn on the drawing sheet is 20 cm



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- 3) Construct a scale of **1.5cm = 1 dm** to read up to **1 meter**. Show on it a length of 0.6m.

Sol) Given data:

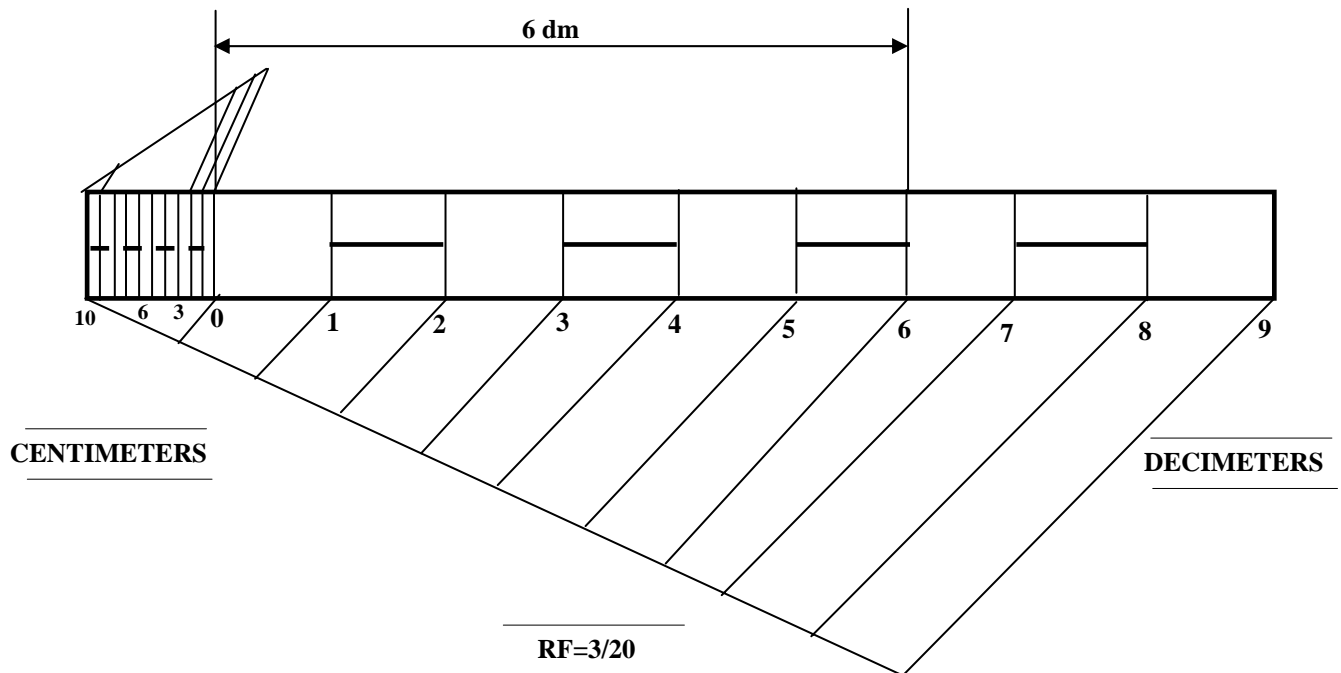
$$\begin{aligned} \text{RF} &= 1.5 \text{ cm} / 1 \text{ dm} \\ &= 1.5 / (1 \times 10) \quad (1 \text{ dm} = 10 \text{ cm}) \\ &= 3/20. \end{aligned}$$

$$\text{Max Length (ML)} = 10 \text{ dm} \quad (1 \text{ m} = 10 \text{ dm}) ; (\text{no. of parts of scale (n)} = 10 \text{ parts})$$

$$\begin{aligned} \text{Length of scale (LOS)} &= \text{RF} \times \text{ML} \quad (\text{in cm}) \\ &= (3/20) \times 1 \times 100 \text{ cm} \quad (1 \text{ m} = 100 \text{ cm}) \\ &= 15 \text{ cm}. \end{aligned}$$

The length of the line that is drawn on the drawing sheet is 15 cm.

The length to be shown is 0.6 m = 6 dm = 6 dm + 0 cm



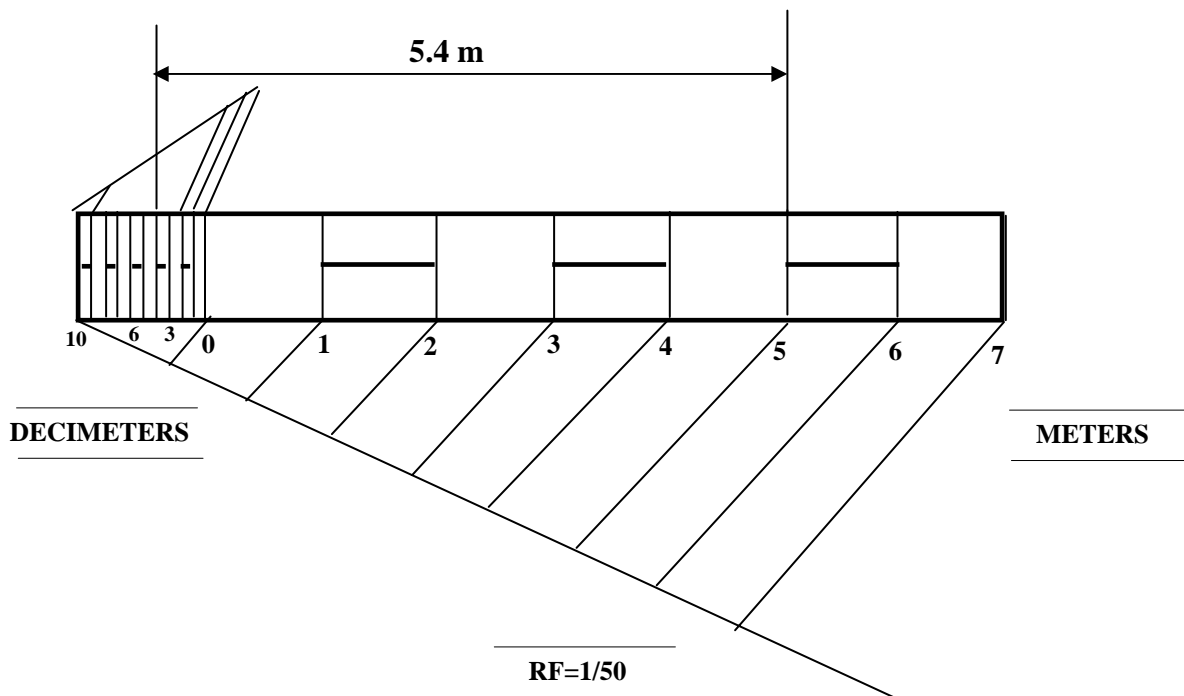
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Q5) Draw a scale of **1:50** showing **meters & decimeters** & to measure up to **8 meters**.
Show the length of **5.4 m** on it.

Sol) Given data:

$$\begin{aligned}
 \text{RF} &= 1/50 \\
 \text{Max Length (ML)} &= 8 \text{ m; (no. of parts of scale (n) = 8 parts)} \\
 \text{Length of scale (LOS)} &= \text{RF} \times \text{ML} \\
 &= (1/50) \times 8 \times 100 \text{ cm (1 m=100 cm)} \\
 &= 16 \text{ cm.}
 \end{aligned}$$

The length of the line that is drawn on the drawing sheet is 16 cm



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- 6) A 3.2 cm long line represents a length of 4 meters. Extend this line to measure lengths up to 25 meters and show on it units of meter and 5 meters. Show the length of 17 meters on this line.

Sol) Given data:

$$\begin{aligned} \text{RF} &= 3.2 \text{ cm} / 4 \text{ m} \\ &= 3.2 / (4 \times 100) \quad (1 \text{ m} = 100 \text{ cm}) \\ &= 1/125. \end{aligned}$$

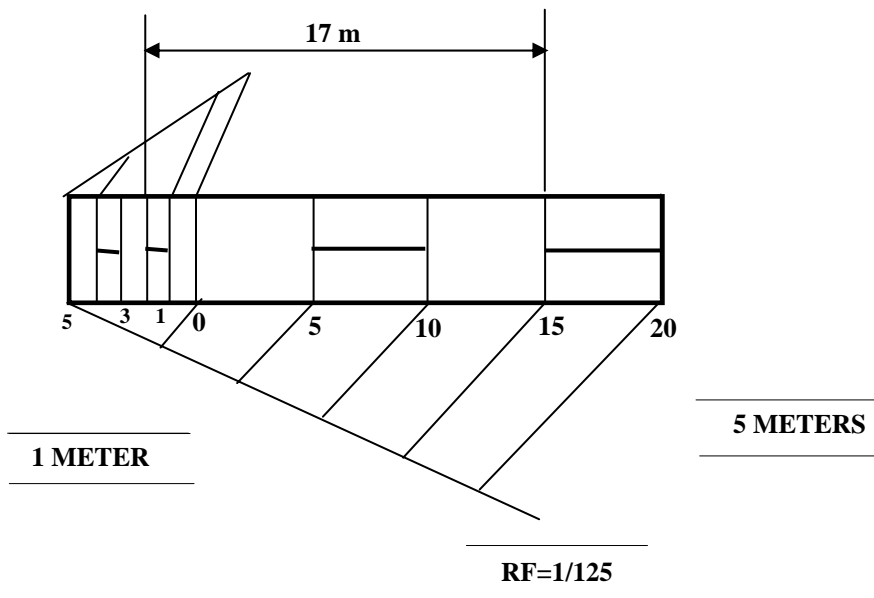
$$\text{Max Length (ML)} = 25 \text{ m; (no. of parts of scale (n) = 5 parts, each is 5 m)}$$

$$\begin{aligned} \text{Length of scale (LOS)} &= \text{RF} \times \text{ML} \quad (\text{in cm}) \\ &= (1/125) \times 25 \times 100 \text{ cm} \quad (1 \text{ m} = 100 \text{ cm}) \\ &= 20 \text{ cm}. \end{aligned}$$

The length of the line that is drawn on the drawing sheet is 20 cm.

The sub scale is to be divided into 5 parts.

The length to be shown is 17 m = 15 m (on main scale) + 2 m (on sub scale)



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27) A room of 1728 m^3 volume is shown by a cube of 216 cm^3 volume. Find the RF & Construct a plain scale to measure up to 42 m. Mark a distance of 22m on this scale

Sol) Given data:

$$\begin{aligned} \text{RF} &= \sqrt[3]{(216) \text{ cm}^3 / (1728 \times (100)^3) \text{ cm}^3} \\ &= 1/200. \end{aligned}$$

Max Length (ML) = 42 m (6 x 7 or 7 x 6);

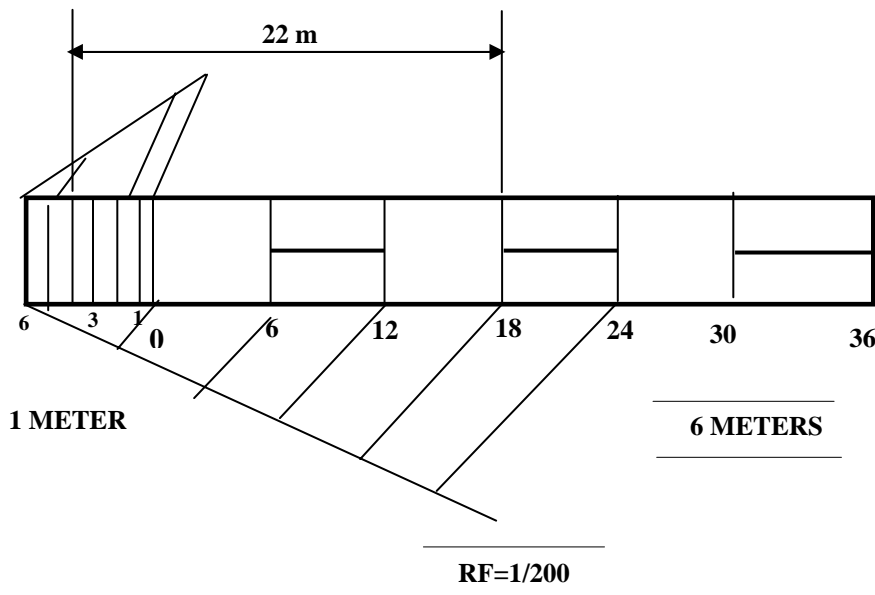
LOS is divided into 6 parts of 7 m each or 7 parts of 6 m each

$$\begin{aligned} \text{Length of scale (LOS)} &= \text{RF} \times \text{ML} \quad (\text{in cm}) \\ &= (1/200) \times 42 \times 100 \text{ cm} \quad (1 \text{ m}=100\text{cm}) \\ &= 21\text{cm}. \end{aligned}$$

The length of the line that is drawn on the drawing sheet is 21 cm.

The sub scale is to be divided into 6 or 7 parts.

The length to be shown is 22 m = 18 m (on main scale) + 4 m (on sub scale)



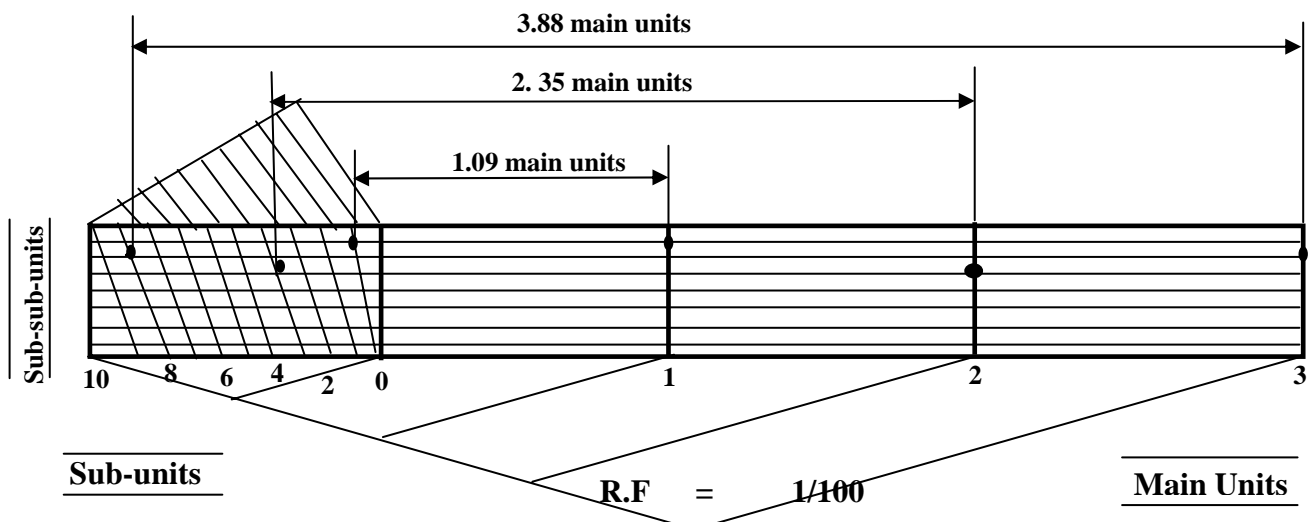
Diagonal Scales

Diagonal scales are used for measurement in three units or a unit and its 2 sub units. It consists of a line (LOS) divided into suitable no. of equal parts (based on Max Length, ML) & the first part is sub-divided into equal parts just as in plain scales. The only extra construction is that on the vertical lines of the scale, the third unit or third subdivision is considered

The zero is placed at the end of 1st main division. Labeling of 0 in main scale is after the first unit. For the vertical divisions, numbering is from bottom to top as 0,1,2,3...

Steps involved in Diagonal Scales:

- 1) Find RF, ML & LOS by using the formula. ($LOS = RF \times ML$).
- 2) Draw the line of length = LOS & divide it into n equal parts based on ML. Draw a rectangle on LOS of height 5 cm.
- 3) Divide the first division also into some equal parts based on the unit and its sub-unit given in the problem & number them as 1,2,3...
- 4) Divide the vertical height also into equal parts depending on the relation between the 2 sub units (Usually 10 parts) & number as 0,1,2,3... from bottom to top.
- 5) Join point 9 of first sub division to the point 10 of the vertical division. The line is inclined and to this draw parallel lines from points 8, 7, 6... up to 0 on the first part sub division.
- 6) Mark the given value of dimension based on problem.



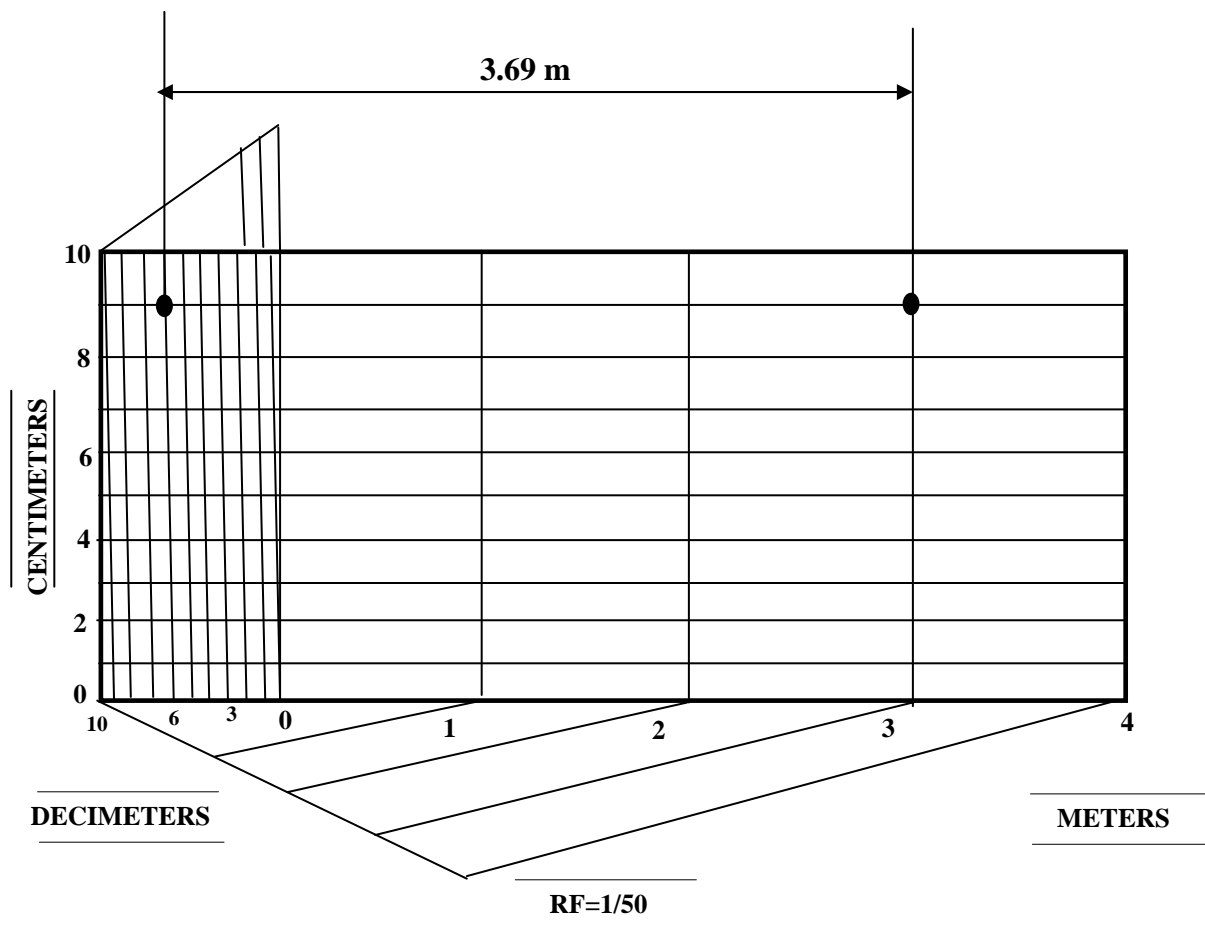
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Q4) Draw a diagonal scale of RF = 3/100, showing meters, decimeters & centimeters & to measure up to 5 meters. Show the length of 3.69 meters on it.

Sol) Given data:

$$\begin{aligned}
 \text{RF} &= 3/100 \\
 \text{Max Length (ML)} &= 5 \text{ m; (no. of parts of scale (n) =5 parts)} \\
 \text{Length of scale (LOS)} &= \text{RF} \times \text{ML} \\
 &= (3/100) \times 5 \times 100 \text{ cm (1 m=100 cm)} \\
 &= 15 \text{ cm.}
 \end{aligned}$$

The length of the line that is drawn on the drawing sheet is 15 cm



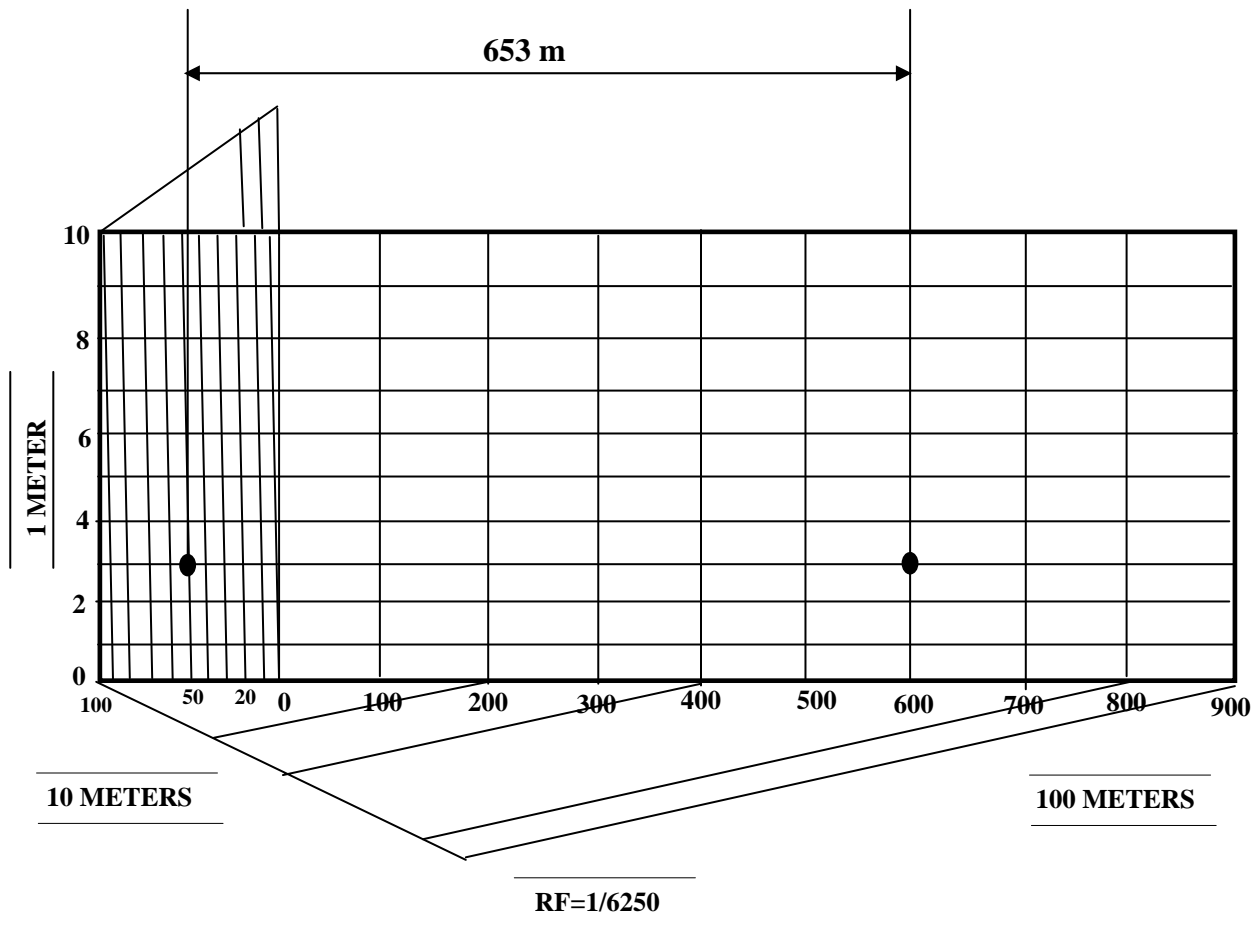
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Q7) Construct a diagonal scale of $RF = 1/6250$ to read up to 1 kilometer and to read meters on it. Show a length of 653 meters on it.

Sol) Given data:

$$\begin{aligned}
 \text{RF} &= 1/6250 \\
 \text{Max Length (ML)} &= 1000 \text{ m (1 km = 1000 m) (10 x 100)} \\
 \text{No. of parts of scale (n)} &= 10 \text{ parts (each of 100 meters)} \\
 \text{Length of scale (LOS)} &= \text{RF X ML} \\
 &= (1/6250) \times 1 \times 1000 \times 100 \text{ cm (1 m=100 cm)} \\
 &= 16 \text{ cm.}
 \end{aligned}$$

The length of the line that is drawn on the drawing sheet is 16 cm



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Q9) An area of 144 sq cm on a map represents an area of 36 sq km on the field. Find the RF of the scale for this map and draw a diagonal scale to show kilometers, hectameters and decameters and to measure up to 10 kilometers. Indicate on the scale a distance of 7 kilometers, 5 hectameters and 6 decameters.

Sol) Given data:

$$RF = \sqrt{144 / (36 \times (1000 \times 100)^2)} = 1/50,000.$$

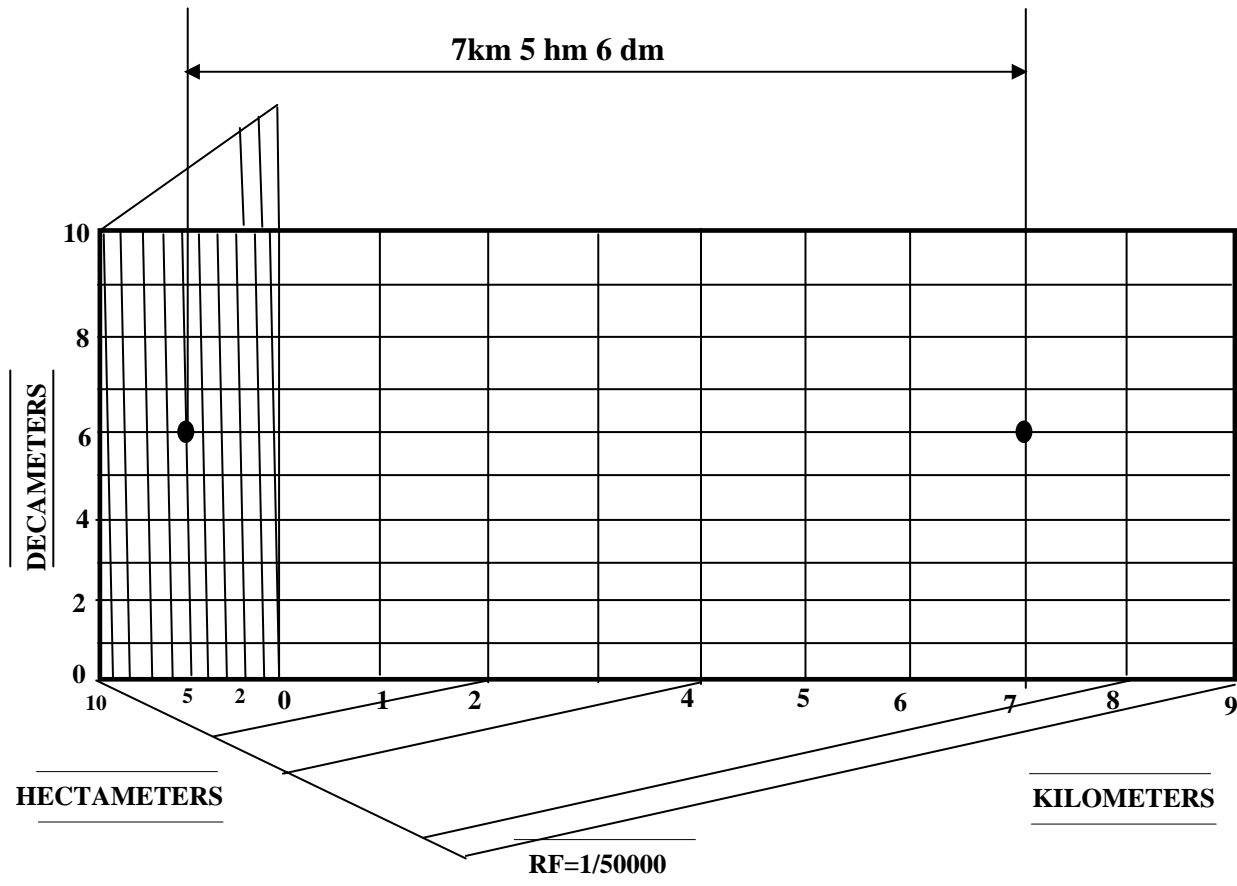
$$\text{Max Length (ML)} = 10 \text{ km.}$$

$$\text{No. of parts of scale (n)} = 10 \text{ parts (each of 1 km)}$$

$$\text{Length of scale (LOS)} = (1/50000) \times 10 \times 1000 \times 100 \text{ cm (1 m=100 cm)}$$

$$= 20 \text{ cm.}$$

The length of the line that is drawn on the drawing sheet is 20 cm. The first division is shown as enlarged for clear understanding.



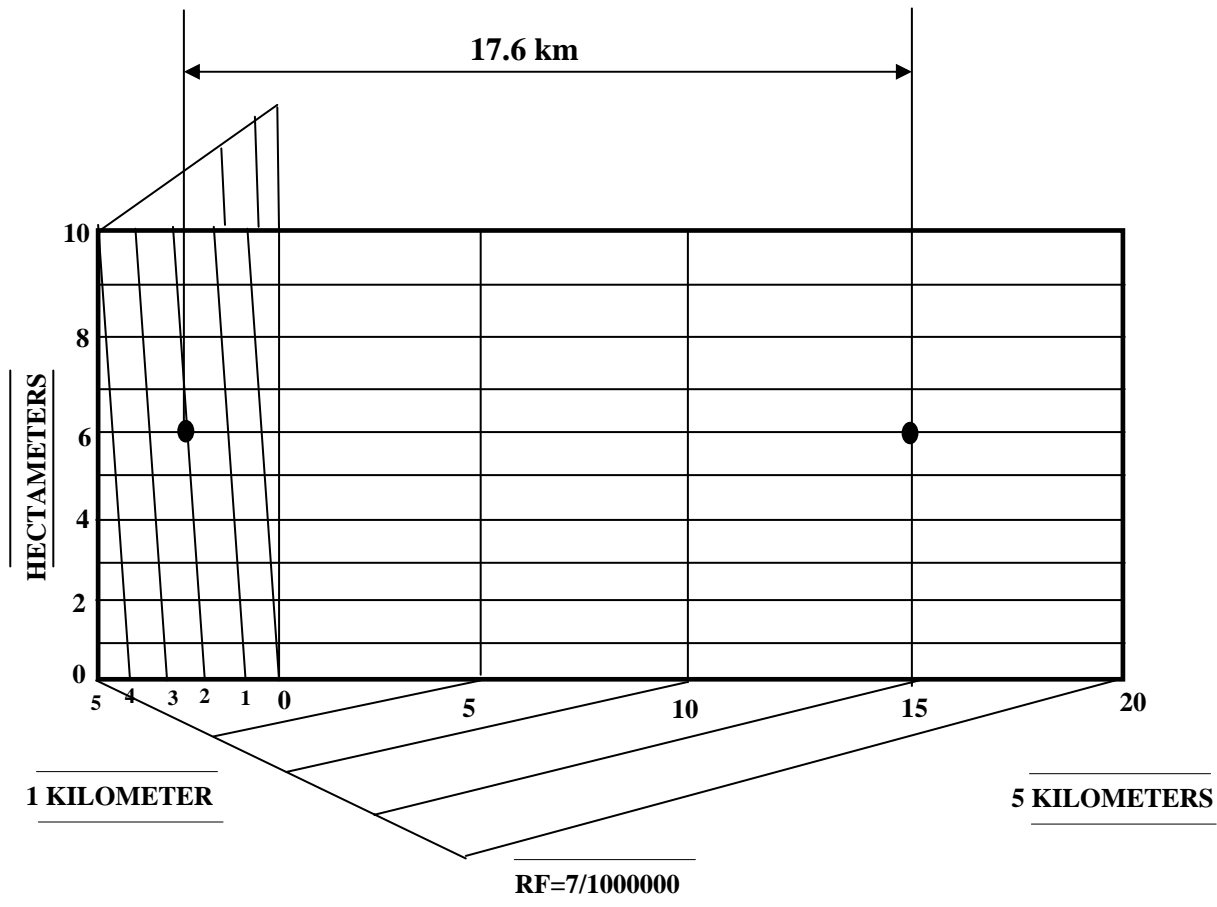
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Q8) On a map, the distance between two points is 14 cm. The real distance between them is 20 km. Draw a diagonal scale of this map to read kilometers and hectameters, & to measure up to 25 km. Show a distance of 17.6 km on this scale.

Sol) Given data:

$$\begin{aligned}
 \text{RF} &= 14 \text{ cm} / (20 \times 1000 \times 100) \text{ cm} = 7/1000000. \\
 \text{Max Length (ML)} &= 25 \text{ km}; (5 \times 5) \\
 \text{(no. of parts of scale = 5 parts, each of 5 km)} \\
 \text{Length of scale (LOS)} &= \text{RF} \times \text{ML} \\
 &= (7/1000000) \times 25 \times 1000 \times 100 \text{ cm (1 km=1000 x 100 cm)} \\
 &= 17.5 \text{ cm.}
 \end{aligned}$$

The sub unit is divided into 5 parts so that each division represents 1 km. The vertical sub-sub unit is hectameter and it is divided into 10 parts as 1 km = 10 hm.



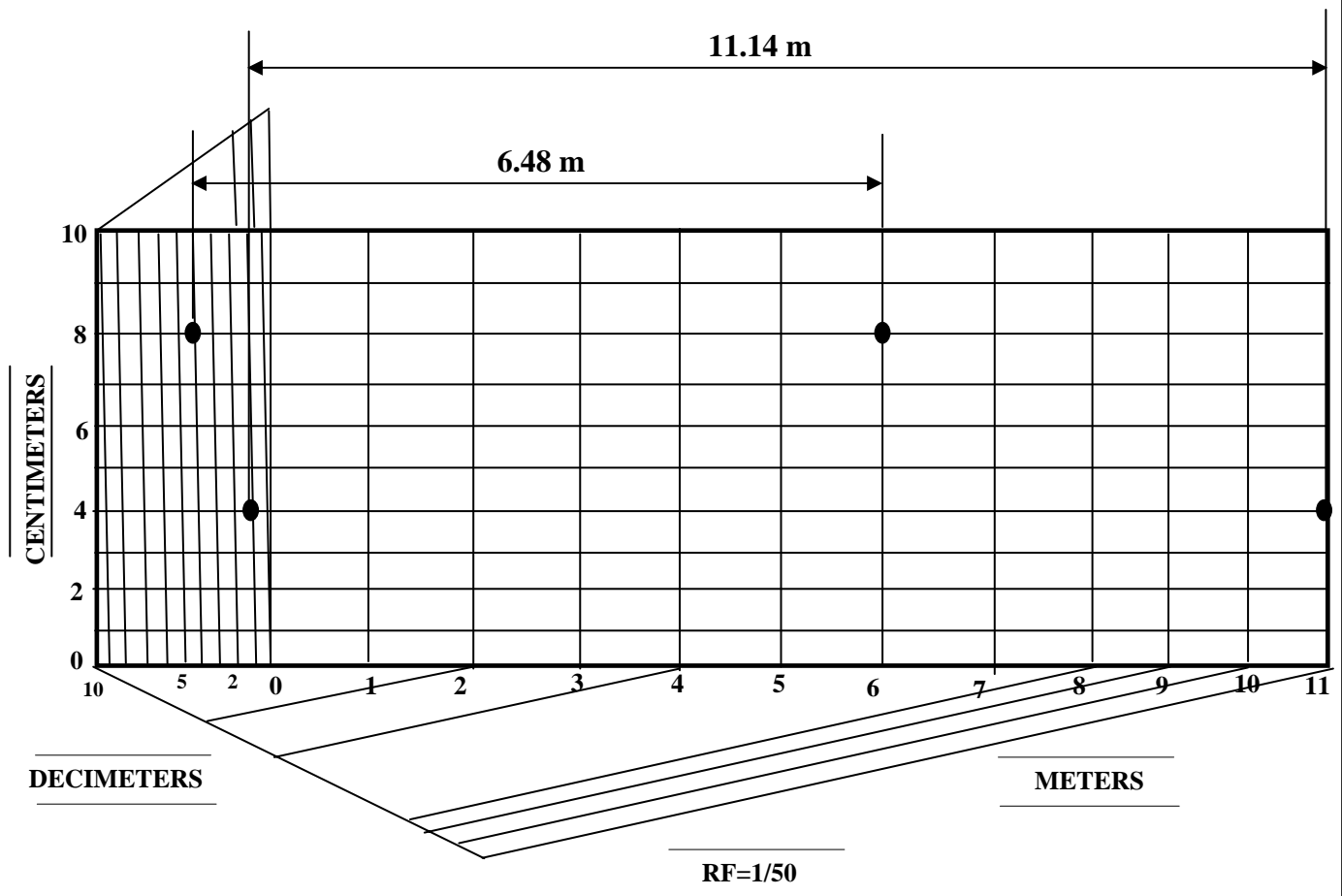
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26) On a building plan, a line 20 cm long represents a distance of 10 m. Device a diagonal scale for the plan to read up to 12 m, showing meters, decimeters and centimeters. Show on your scale the lengths 6.48 m & 11.14 m.

Sol) Given data:

$$\begin{aligned}
 \text{RF} &= 20 / (10 \times 100) = 1/50. \\
 \text{Max Length (ML)} &= 12 \text{ m.} \\
 \text{No. of parts of scale (n)} &= 12 \text{ parts (each of 1 m)} \\
 \\
 \text{Length of scale (LOS)} &= (1/50) \times 12 \times 100 \text{ cm (1 m=100 cm)} \\
 &= 24 \text{ cm.}
 \end{aligned}$$

The length of the line that is drawn on the drawing sheet is 24 cm. The first division is shown as enlarged for clear understanding.



Vernier Scales

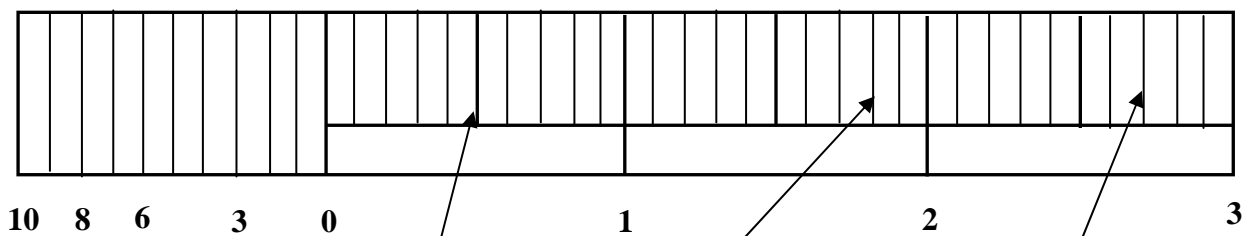
- ➔ Vernier scales are used for measurement in three units or a unit and its 2 sub units.
- ➔ (i) It consists of a line (**length=LOS**) divided into suitable no. of equal parts (Based on **Max Length, ML**).
- ➔ (ii) (a) The first part is sub-divided into equal parts just as in plain scales.
(b) Also, the main scale has to be divided into minor equal parts (usually of 10 parts) to measure the lengths in first decimal or 10's place.
- ➔ (iii) The only extra construction is that above the first sub-division, a separate Vernier scale is drawn which is used for measuring the **sub-sub-units**.

Construction of a vernier scale:

Consider the usual plain scale problem.
We shall learn how to construct a vernier scale on it.

Steps:

- 1) Find the RF, ML & LOS. Draw the scale & mark the divisions of plain scale and sub scale (all these steps are same as in plain scales).
- 2) On every main scale, divide into equal sub divisions between 0-1, 1-2, 2-3 & give the numbering accordingly.



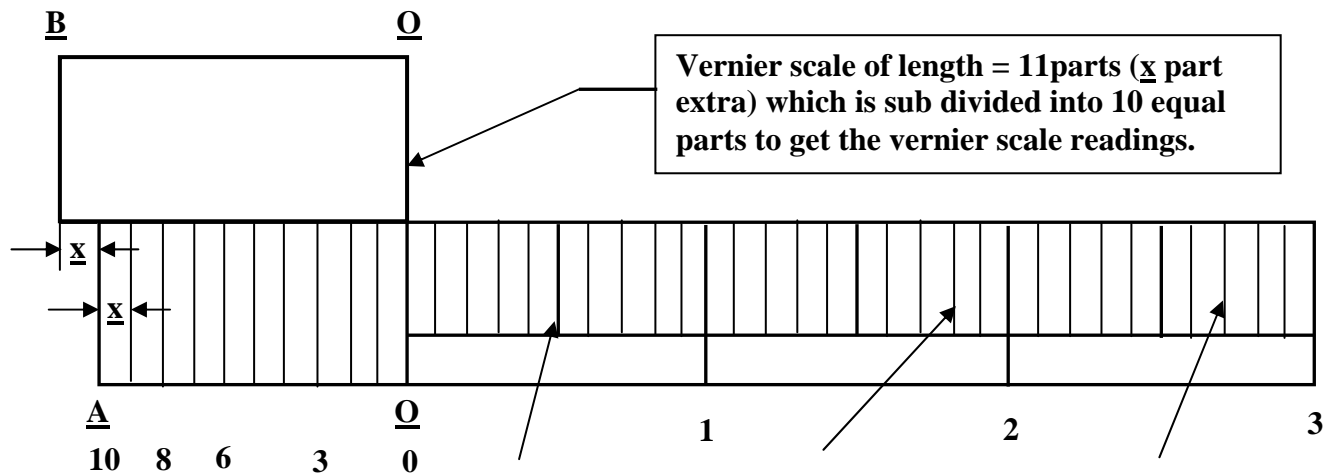
Observe that all main divisions are again sub-divided into 10 equal parts so that measurements like 0.5, 1.8, 2.7, etc can be made on main scale itself. This is not done in plain scales or diagonal scales but only in vernier scales.

Vernier Scales...Steps..Continued...

- 3) Now after the main scale and sub scale divisions have been completed, we have to draw the vernier scale on the top of the first sub-division between 10 & 0.

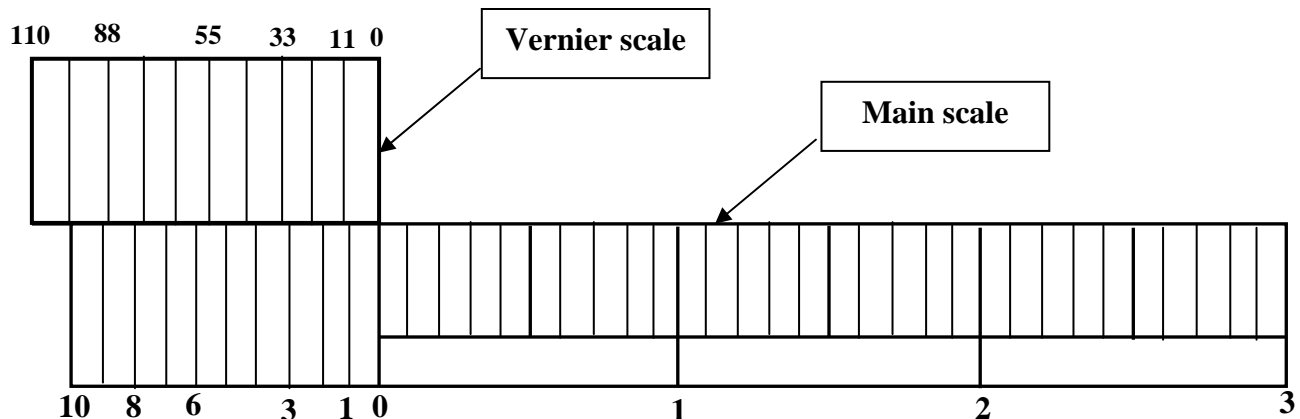
Procedure for vernier scale above the first sub-division:

- 4) Let the length of each division of AO be x. Above the first sub-division AO, extend the line by x & draw a box BO of 5 mm height & of length = AO + x.



- 5) Now BO should be divided into 10 equal parts to get the vernier scale.

Each division on this vernier scale = $(BO)/10 = (AO+x)/10 = (10+1)/10 = 1.1$ sub units or 11 sub-sub-units. (E.g.: if meters, decimeters & centimeters are the units considered in the problem, then each **Vernier scale reading will be multiples of 11 cm or 1.1 dm**)



Measurements in Vernier Scales

- ➔ In Vernier scales, it is not possible to measure the sub-division directly. Only 2 types of readings can be taken. They are the main scale reading and the vernier scale reading.
- ➔ As the vernier scale reading consists of multiples of 11, subtract the multiple of 11 from the main scale reading which has the same last digit.

For E.g. 2.64 decimeters is to be measured.

Since 4 is the last digit, subtract 0.44 from it. Hence **2.64=0.44+2.20**.

E.g.(1)

$$\begin{array}{r}
 2.64 \\
 - 0.44 \\
 \hline
 2.20
 \end{array}$$

Vernier Scale reading

Main Scale reading

E.g.(2)

Measure 3.58

$$\begin{array}{r}
 3.58 \\
 - 0.88 \\
 \hline
 2.70
 \end{array}$$

Vernier Scale reading

Main Scale reading

E.g. (3)

$$\begin{array}{r}
 0.64 \\
 - 0.44 \\
 \hline
 0.20
 \end{array}$$

Vernier Scale reading

Main Scale reading

E.g. (4)

0.48 →

$$\begin{array}{r}
 0.48 \\
 - 0.88 \\
 \hline
 -0.40
 \end{array}$$

Vernier Scale reading

Main Scale reading; to be marked from 0 to 4 to the left of the first subdivision on AO

Exercise Problems

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- 21) Construct a scale of **R.F = 1/(2.5)** to show decimeters & centimeters and by a **vernier** to read millimeters, to **measure** up to 4 decimeters. Show on it lengths **2.34 dm, 1.42 dm & 0.38 dm**.

Sol) Given data:

$$RF = 1/125.$$

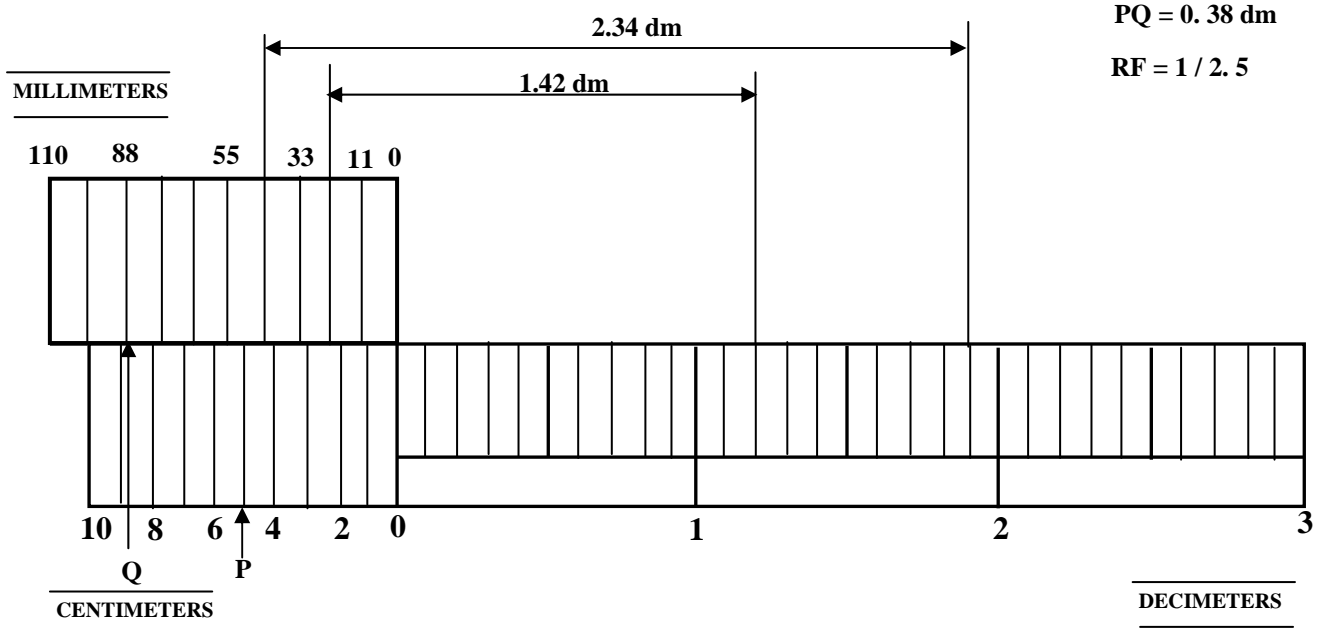
Max Length (ML) = 4 dm; (no. of parts of scale (n) =4 parts, each is 1 dm)
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$$\begin{aligned} \text{Length of scale (LOS)} &= RF \times ML \quad (\text{in cm}) \\ &= (1/2.5) \times 4 \times 10 \text{ cm} \quad (1 \text{ dm}=10\text{cm}) \\ &= 16 \text{ cm}. \end{aligned}$$

The length of the line that is drawn on the drawing sheet is 16 cm & divided into 4 parts.

The sub scale is to be divided into 10 parts & vernier scale also into 10 parts.
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The lengths are 2.34 (0.44, 1.90), 1.42 (0.22, 1.20) & 0.38 (0.88, -0.50) (VSD, MSD) (VSD, MSD) (VSD, MSD)



Exercise Problems

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29) The actual length of 500m is represented by a line of 15cm on a drawing. Construct a vernier scale to read up to 400 m. Mark on the scale a length of 349m.

Sol) Given data:

$$RF = \frac{15 \text{ cm}}{(500 \times 100) \text{ cm}} = \frac{3}{10000}$$

$$\text{Max Length (ML)} = 400 \text{ m; (no. of parts of scale (n) = 4 parts, each is 100 m)}$$

$$\begin{aligned} \text{Length of scale (LOS)} &= RF \times ML \quad (\text{in cm}) \\ &= \left(\frac{3}{10000}\right) \times 400 \times 100 \text{ cm} \quad (1 \text{ m}=100\text{cm}) \\ &= 12 \text{ cm.} \end{aligned}$$

The length of the line that is drawn on the drawing sheet is 12 cm & divided into 4 parts.

The sub scale is to be divided into 10 parts & vernier scale also into 10 parts.

The length is 349 m (99+ 250); 99 on vernier scale & 250 on main scale.
(VSD, MSD)

